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Abstract Title: The "ARLS" Automatic Regularized Least Squares Package

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Introduction

We are interested in completely automatic Tikhonov regularization of linear systems, Ax=b.

Objective

In our context we focus on the following, where PCV means the "Picard Condition" Vector.

$Ax=b \rightarrow USV^Tx = b \rightarrow V^Tx = S^+U^Tb \rightarrow V^Tx = PCV$

With inverse problems PCV usually declines, turns around, and rises to millions. (We are only interested in the magnitude of the elements of PCV, not the sign.) The Picard Condition says that analysts should not expect a good solution unless the PCV declines. Our goal is to make that happen in an effective way for many problems.

Methods

Our new work finds the smallest lambda that makes the logarithm of the PCV "decline", where by "decline" we mean that a curve fit to the (regularized) PCV has negative slope.

Results

The resulting algorithm is stunningly effective, and can be the basis for a variety of constrained solutions, including NNLS's nonnegative constraint. However, ARLS does not have the possibility of a "failed to converge" diagnostic as NNLS does.

Significance

The fact that ARLS is so reliable is a surprising development. No software for ill-conditioned linear systems I know of has ever been this reliable, and thus able to be the basis for extensive constraint functions, though NNLS has been close, and a very good resource for many years for the specific case of nonnegative constraints.

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References

Hansen, P.C. The discrete picard condition for discrete ill-posed problems. *BIT* **30**, 658–672 (1990). https://doi.org/10.1007/BF01933214